**Notes: Week of Sept21.Fall2012**

Course website: [www.cis.syr.edu/~sueo/cis275](http://www.cis.syr.edu/~sueo/cis275)

**Common Terms in Mathematics:**

* Definition
* Postulate (or axiom)
* Theorem
* Proof
* Proposition
* Lemma
* Corollary
* Claim
* Conjecture
* Counterexample

**Definitions**

Definition – in mathematics, a statement that stipulates the meaning of a new term, symbol or object

Specifies precisely what is meant by the term

Serves as the sole authority of what the term means

Any ⊆sequent statements about that term take their meaning from the definition

Standard form of a mathematics definition:

*[Object]* x is *[defined term]* if *[defining property about x]*

Postulate (axiom) – a statement assumed true without proof

* Typically very basic, fundamental statements about objects
* Serve as starting points for deriving other statements
* Ie: If n is a natural number, then n+1 is also a n

Theorem – a statement that follows logically from axioms or other statements that have already been established

To be called a theorem a statement must have proof – a valid argument based on axioms, definitions and proven theorems.

Lemma – A theorem used to prove another theorem

Proposition – used to refer to a theorem that is considered less significant than other theorems

Corollary – theorem that follows immediately from another theorem via a very short argument

Claim – statement that we intend to prove

Conjecture – statement thought to be true but has not been proved

Counterexample –a value that shows a statement to be false

**Mathematical Proofs**

Tend to be expressed in paragraph form

May not spell out all details (depending on audience)

Certain features of a proof:

Collection of hypotheses (or premises)

A desired conclusion

The need to show that whenever all hypotheses are true, the conclusion is also true

**Format for proofs for CIS275:**

Specify what you’re trying to prove: Proposition: …

Label the start and indicate the method: Proof: (direct)

Explicitly state any initial assumptions *Let m be an integer, and suppose that m Is even.*

State what you need to show Need to show: *M+1 is odd, that is,…*

Fill gaps with the heart of the proof

Wrap up the proof: Because m was arbitrary, the proposition is true

*Def’n – An integer is even iff ∃k ∈ ℤ such that n=2k*

*Def’n – An integer is odd iff ∃k ∈ ℤ such that n =2k+1*

*Facts: 1 – The sum of two integers is an integer*

*2- the product of two integers is an integer*

**Claim**: Suppose m and n are integers. If m and n are both odd, then m\*n is also odd

**Proof** (direct)

Let m and n be arbitrary integers and suppose that m and n are both odd

[**NTS(Need to show) :** m\*n is odd (which means ∃k ∈ ℤ such that m\*n=2k+1)]

By definition of odd, there exist integers h,j such that m=2h+1, n=2j+1

By algebra, m\*n = (2h+1)(2j+1) = 4hj +2h +2j +1 = 2(2hj +j + h) +1

Because h and j are integers, 2hj + h + j is an integer

Thus there exists an integer k such that m\*n = 2k+1, so m\*n is odd.

Because m\*n were arbitrary, the claim is true.

Suppose that we want to prove W ⊆ Z

**Definition:** W ⊆ Z iff ∀ x in U, if x ∈ W then x ∈ Z

Therefore to show W ⊆ Z, Consider arbitrary x ∈ W and show x ∈ Z

**Claim:** For all sets A and B, A ∩ B is a subset of A ∪ B

**Proof:** Let A,B be arbitrary sets, and suppose x ∈ A ∩ B, [NTS: x ∈ A U B ]

By defn of Intersect, x ∈ A and x ∈ B, since x ∈ A, by defn of U, x is in A U B

**Since x was arbitrary, A ∩ B is a subset of A ∪ B**

Thursday:

To prove ∀ x ∈ U P(x) Q(x)

1. Consider arbitrary a ∈ U
2. Supposed that P(a) is true
3. Show that Q(a) is true

Special Instance: To show W ⊆ Z

Consider arbitrary x element of W

Show x element of Z

**Claim:** Let A, B, C be sets. If A ⊆ B and B ⊆ C, then A ⊆ C

[**NTS**: A ⊆ C. ∀ x, x if x ∈ A then x ∈ C]

**Proof:**

Consider arbitrary x element of A

Because A ⊆ B, x ∈ B. Because B ⊆ C, x ∈ C.

**Because x was arbitrary, A is a subset of C**

**Claim:** Let A B C D be sets . If A ∈ B and C ⊆ D, then A x C ⊆ B x D

**Proof:**

Suppose A ⊆ B and C ⊆ D

[**NTS**: AxC ⊆ B x D]

Consider arbitrary (a,c) ∈ A x C

Because A ⊆ B , a ∈ B, because C ⊆ D, c ∈ D

Thus (a,c) ∈ B x D

**Because (a,c) was arbitrary, AxC ⊆ BxD**

**Claim:** Let A,B be sets. If A ⊆ B, then P(A) ⊆ P(B)

**Proof:**

Suppose A ⊆ B

[**NTS**: P (A) ⊆ P (B)]

Consider arbitrary X ∈ P(A)

By Defn of power set, X ⊆A

By subset transitivity A ⊆ B which means X ∈ P (B)

**Because X was arbitrary P (A) ⊆ P (B)**

**Claim:** Let A,B,C be sets. Then C ⊆ A ∩ B iff C ⊆ A and C ⊆ B

**Proof** There are two conditionals to prove.

1. If C ⊆ A ∩ B and then C ⊆ A and C ⊆ B
2. If C ⊆ A and C ⊆ B then C ⊆ A ∩ B

To Prove 1: suppose C ⊆ A ∩ B

[NTS: C ⊆ A and C ⊆ B

Consider arbitrary x ∈ C. Since C ⊆ A ∩ B…x ∈ A ∩ B, which means x ∈ A and x ∈ B

Since x was arbitrary C ⊆ A and C ⊆ B

To Prove 2- suppose C ⊆ A and C ⊆ B [NTS: C ⊆ A∩B]

Consider arbitrary x in C, By defn of subset, x ∈ A and x ∈ B thus x ∈ A ∩ B

Because x was arbitrary C ⊆ A ∩ B

To PROVE BICONDITIONALS: First prove A → B then prove B → A

Recall that for sets A,B subsets of U

A=B iff ∀ x ∈ U, (x ∈ A ≡ x ∈ B)

So to prove A = B, consider arbitrary x, show biconditional. To show biconditional, show both conditionals

Claim: Let U be universal set. For all sets A, (Ac)c = A

Proof Consider arbitrary set A, show that for all x in U, x is in Acc iff x in A

[OUTLINE]

Consiter arbitrary x in U,

X ∈ (Ac)c ≡ x ∉ a c

X ∈ (Ac)c  ≡ ¬ (x ∈ Ac )

X ∈ (Ac)c ≡ ¬ (x ∉ A)

X ∈ (Ac)c ≡ ¬ (¬ x ∈ A)

X ∈ (Ac)c ≡ x in A

[*outline doesn’t count as proof*]

Thus x ∈ (Ac)c ≡ x ∈ a is true because x was arbitrary. That is a good outline for a proof

Proof: consider arbitrary set A

Nts (Ac)c ⊆ A and A ⊆ (Ac)c

To prove 1: consider arbitrary x ∈ (Ac)c

By defn (Ac)c, x not in Ac. Thus it is not the case that x in Ac which means it is not the case that x ∉ A

Therefore x ∈ A and because x was arbitrary Hence (Ac)c ⊆ A

To prove 2: consider arbitrary x ∈ A

By definition A, x ∉ Ac. Thus it is not the case that x is in Ac meaning it is not the case that x is in A

Therefore x ∈ (Ac)c and because x was arbitrary Hence A ⊆ (Ac)c

By proofs 1 and 2, (Ac)c ⊆ A and A ⊆ (Ac)c

**Claim:** Let A, B be sets

If A ⊆ A ∩ B and B ⊆ A ∩ B then A = B

**Proof:** suppose A ⊆ A ∩ B and B ⊆ A ∩ B [NTS: A = B ]

To prove A ⊆ B consider arbitrary x ∈ A , by defn ⊆, x ∈ A ∩ B, which means x ∈ B. Thus since x arbitrary A ⊆ B

To prove B ⊆ A. consider arbitrary x ∈ B. because B ⊆ A ∩ B, x ∈ A ∩ B

Which means x ∈ A since x arbitrary, B ⊆ A

Since A ⊆ B and B ⊆ A, we know A = B